

(b) Determine a, b, c such that

$$\lim_{\theta \rightarrow 0} \frac{\theta(a + b \cos \theta) - c \sin \theta}{\theta^5} = 1.$$

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Show that $\frac{1}{1.1.3} + \frac{1}{2.3.5} + \frac{1}{3.5.7} + \dots \infty = 2 \log 2 - 1.$

17. Transform the equation

$$x^4 - 4x^3 - 18x^2 - 3x + 2 = 0$$
 into an equation with the third term absent.

18. Find the eigen values and eigen vectors of the

$$\text{matrices } A = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}.$$

19. Solve the equation $3x + y + 2z = 3$, $2x - 3y - z = 3$, $x + 2y + z = 4$ by using Cramer's rule.

20. Show that

$$128 \sin^8 \theta = \cos 8\theta - 8 \cos 6\theta + 28 \cos 4\theta - 56 \cos 2\theta + 3$$

Time : Three hours

Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL questions.



Find the coefficient of x^n in the expansion of e^{a+bx} .

Show that if $x > 0$,

$$\log x = \frac{x-1}{x+1} + \frac{1}{2} \cdot \frac{x^2-1}{(x+1)^2} + \frac{1}{3} \cdot \frac{x^3-1}{(x+1)^3} + \dots \infty$$

3. Define Reciprocal equation.

4. If α is a root of the equation $x^3 + x^2 - 2x - 1 = 0$ show that $\alpha^2 - 2$ is also a root.

5. Find the eigen value of $\begin{bmatrix} a & h & z \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}.$

6. Define Skew-Hermitian matrices with example.

7. If $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, find A^T .

8. Find the determinant of $\begin{bmatrix} 3 & 2 \\ 5 & 1 \end{bmatrix}$.

9. Expand $\tan 7\theta$ in terms of $\tan \theta$.

10. Evaluate $\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{\sin^3 x}$.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Resolve into partial fractions $\frac{x^2 + 1}{(2x - 3)(x - 1)^2}$.

Or

(b) Find the sum to infinity of the series

$$1 + \frac{2}{6} + \frac{2.5}{6.12} + \frac{2.5.8}{6.12.18} + \dots \infty$$

12. (a) Solve the equation

$$4x^4 - 20x^3 + 33x^2 - 20x + 4 = 0.$$

Or

(b) Diminish by 2 the roots of the equation

$$x^4 + x^3 - 3x^2 + 2x - 4 = 0.$$

13. (a) Verify Cayley-Hamilton theorem for

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}.$$

Or

(b) Show that the matrix $A = \begin{bmatrix} a + ic & -b + id \\ b + id & a - ic \end{bmatrix}$ is unitary if and only if $a^2 + b^2 + c^2 + d^2 = 1$.

14. (a) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 2 & 0 \\ 3 & 1 & 5 \\ 4 & -2 & 1 \end{bmatrix}$ find

$A + B$ and $A - B$.

Or

(b) Find the inverse of $A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & 2 \\ 3 & 2 & 4 \end{bmatrix}$ using the adjoint method.

15. (a) Prove that

$$\cos 6\theta = 32 \cos^6 \theta - 48 \cos^4 \theta + 18 \cos^2 \theta - 1.$$

Or